

FACOLTÀ DI SCIENZE MM.FF.NN.
Corso di laurea in Scienze Biologiche

Prova scritta di Matematica
del 18-02-2010

Cognome..... Nome..... N° Matricola.....

- OK 1. Calcolare il seguente limite:

$$\lim_{x \rightarrow 0} \left[\frac{2 - 2 \cos^2 x}{x^2} + (x^2 + 1) \right].$$
 svolto

2. Enunciare, dimostrare e commentare il teorema di Fermat.

3. Calcolare il seguente integrale indefinito:

$$\int e^{2x} \sin(3x) dx.$$
 X DA FERMAT svolto

- OK 4. Calcolare la derivata della seguente funzione:

$$f(x) = 2x \cos(\lg x) + 3x \sin(\log x) + 2x.$$

5. Studiare la seguente funzione:

$$f(x) = (x+2) e^{-\frac{1}{5}x^2} + 1.$$

*svolto con l'analisi
calcolo*

Svolgere come

*$(2+x) e^{-\frac{1}{5}x^2} + 2$ dell'altro compito
stessa data*

$$\lim_{x \rightarrow 0} \frac{2 - 2 \cos^2 x}{x^2} + x^2 + 1 = \lim_{x \rightarrow 0} \frac{2 - 2 \cos^2 x}{x^2} + \lim_{x \rightarrow 0} x^2 + 1 =$$

$$= \lim_{x \rightarrow 0} \frac{2 - 2 \cdot \cos^2 0}{0} + \lim_{x \rightarrow 0} 0^2 + 1 = \lim_{x \rightarrow 0} \frac{2 - 2 \cdot 1}{0} + 1 = \frac{2 - 2}{0} + 1 = \frac{0}{0} + 1$$

$\cos \phi = 1$

$\downarrow$
f. ind.

Risolvere la forma indeterminata $\frac{0}{0}$

$$\lim_{x \rightarrow 0} \frac{2 - 2 \cos^2 x}{x^2} = \lim_{x \rightarrow 0} \frac{2(1 - \cos^2 x)}{x^2} = \lim_{x \rightarrow 0} \frac{2(\sin^2 x)}{x^2} =$$

$1 - \cos^2 x = \sin^2 x$

$$= 2 \cdot \lim_{x \rightarrow 0} \frac{\sin^2 x}{x^2} = 2 \cdot \lim_{x \rightarrow 0} \frac{\sin x}{x} \cdot \frac{\sin x}{x} = 2 \cdot \lim_{x \rightarrow 0} \frac{\sin x}{x} \cdot \lim_{x \rightarrow 0} \frac{\sin x}{x} =$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$= 2 \cdot 1 \cdot 1 = 2$$

Allora

$$\lim_{x \rightarrow 0} \frac{2 - 2 \cos^2 x}{x^2} + x^2 + 1 = 2 + 0 + 1 = 3$$

VERIFICATO

$$\lim_{x \rightarrow 0} \frac{2 - 2 \cos^2 x}{x^2} = \frac{0}{0} \stackrel{H}{=} \frac{+2 \cdot 2 \cos x \cdot \sin x}{2x} = \frac{2 \cos x \sin x}{x} = \frac{0}{0} \stackrel{H}{=} \frac{2(-\sin^2 x + \cos^2 x)}{1}$$

$$= 2 \cdot \frac{(0 + 1)}{1} = 2$$

$$\int e^{2x} \sin 3x \, dx = \left\| \begin{array}{l} \text{PER PARTI} \\ \text{RECURSIVO} \end{array} \right. \quad \text{obs} \quad \int e^{2x} \, dx = \frac{e^{2x}}{2} \quad \text{infatti} \quad \left(\frac{e^{2x}}{2} \right)' = \frac{1}{2} e^{2x} \cdot 2 = e^{2x}$$

$$= \frac{e^{2x}}{2} \sin 3x - \int \frac{e^{2x}}{2} \cdot (\cos 3x \cdot 3) \, dx =$$

$$= \frac{e^{2x}}{2} \sin 3x - \frac{3}{2} \int e^{2x} \cos 3x \, dx =$$

$$= \frac{e^{2x}}{2} \sin 3x - \frac{3}{2} \left[\frac{e^{2x}}{2} \cos 3x - \int \frac{e^{2x}}{2} (-\sin 3x) \cdot 3 \, dx \right] =$$

$$= \frac{e^{2x}}{2} \sin 3x - \frac{3}{2} \left[\frac{e^{2x}}{2} \cos 3x + \frac{3}{2} \int e^{2x} \sin 3x \, dx \right] =$$

$$= \frac{e^{2x}}{2} \sin 3x - \frac{3}{2} \cdot \frac{e^{2x}}{2} \cos 3x - \frac{3}{2} \cdot \frac{3}{2} \int e^{2x} \sin 3x \, dx =$$

$$= \frac{e^{2x}}{2} \sin 3x - \frac{3}{4} e^{2x} \cos 3x - \frac{9}{4} \int e^{2x} \sin 3x \, dx$$

HO ATTENZIONE CHE

$$\underline{\int e^{2x} \sin 3x \, dx} = \frac{e^{2x}}{2} \sin 3x - \frac{3}{4} e^{2x} \cos 3x - \frac{9}{4} \underline{\int e^{2x} \sin 3x \, dx} \Rightarrow$$

$$\int e^{2x} \sin 3x \, dx + \frac{9}{4} \int e^{2x} \sin 3x \, dx = \frac{e^{2x}}{2} \sin 3x - \frac{3}{4} e^{2x} \cos 3x$$

$$\frac{13}{4} \int e^{2x} \sin 3x \, dx = \frac{e^{2x}}{2} \sin 3x - \frac{3}{4} e^{2x} \cos 3x \Rightarrow$$

$$\Rightarrow \int e^{2x} \sin 3x \, dx = \frac{4}{13} \left(\frac{e^{2x}}{2} \sin 3x - \frac{3}{4} e^{2x} \cos 3x \right) =$$

$$= \frac{4}{13} \frac{e^{2x}}{2} \sin 3x - \frac{4}{13} \frac{3}{4} e^{2x} \cos 3x =$$

$$= \frac{2}{13} e^{2x} \sin 3x - \frac{3}{13} e^{2x} \cos 3x + C \quad \checkmark$$

$$\frac{1+\frac{9}{4}}{\frac{4}{4}} = \frac{4+9}{4} = \frac{13}{4}$$

VERIFICATO

DERIVATIVE

$$y = 2x \cos(\log x) + 3x \sin(\log x) + 2x$$

$$\begin{aligned} y' &= (2x)' \cos(\log x) + 2x (\cos(\log x))' + (3x)' \sin(\log x) + 3x (\sin(\log x))' + (2x)' \\ &= 2 \cos(\log x) + 2x (-\sin(\log x) \cdot (\log x)') + 3 \sin(\log x) + 3x (\cos(\log x) \cdot (\log x)') + 2 \\ &= 2 \cos(\log x) - \frac{2x \sin(\log x)}{x} + 3 \sin(\log x) + \frac{3x \cos(\log x)}{x \log 10} + 2 = \\ &= 2 \cos(\log x) - 2 \sin(\log x) + 3 \sin(\log x) + \frac{3 \cos(\log x)}{\log 10} + 2 = \\ &= 2 \cos(\log x) - 2 \sin(\log x) + 3 \sin(\log x) + \frac{3}{\log 10} \cos(\log x) + 2 \end{aligned}$$

$$y = (x+2) e^{-\frac{1}{6}x^2} + 1$$

$$D:]-\infty; +\infty[$$

SEGN

$$(x+2) e^{-\frac{1}{6}x^2} + 1 \geq 0 \quad \text{non si può dire direttamente}$$

$$\parallel \quad x=0 \Rightarrow y = (2) e^{-\frac{1}{6} \cdot 0} + 1 = 2 + 1 = 3$$

$$P(0, 3)$$

LIMITI

$$\lim_{x \rightarrow -\infty} (x+2) e^{-\frac{1}{6}x^2} + 1 = +\infty \cdot e^{-\infty} + 1 = +\infty \cdot 0 + 1$$

↓
forma indeterminata

$$\lim_{x \rightarrow -\infty} (x+2) e^{-\frac{1}{6}x^2} = \lim_{x \rightarrow -\infty} \frac{x+2}{e^{\frac{1}{6}x^2}} = \frac{-\infty}{+\infty} \stackrel{H}{=} \frac{1}{e^{\frac{x^2}{6} \cdot \frac{2x}{6}}} = \lim_{x \rightarrow -\infty} \frac{1}{\frac{1}{3} e^{\frac{x^2}{3} \cdot x}} = \frac{1}{+\infty(+\infty)} = 0$$

$$\Rightarrow \lim_{x \rightarrow -\infty} (x+2) e^{-\frac{1}{6}x^2} + 1 = 1 \quad \text{A.O.}$$

$$\lim_{x \rightarrow +\infty} (x+2) e^{-\frac{1}{6}x^2} + 1 = +\infty \cdot 0 + 1 = \lim_{x \rightarrow +\infty} \frac{(x+2)}{e^{\frac{x^2}{6}}} + 1 = \frac{\infty}{\infty} + 1 \stackrel{H}{=}$$

$$= \lim_{x \rightarrow +\infty} \frac{1}{e^{\frac{x^2}{6} \cdot \frac{2x}{6}}} + 1 = \lim_{x \rightarrow +\infty} \frac{1}{\frac{1}{3} e^{\frac{x^2}{3} \cdot x}} + 1 = 1 \quad \text{A.O.}$$

DERIVATA PRIMA

$$y' = e^{-\frac{1}{6}x^2} + (x+2) e^{-\frac{1}{6}x^2} \cdot \left(-\frac{1}{3}\right) \cdot 2x = e^{-\frac{1}{6}x^2} \left(1 + (x+2) \cdot \left(-\frac{x}{3}\right)\right) =$$

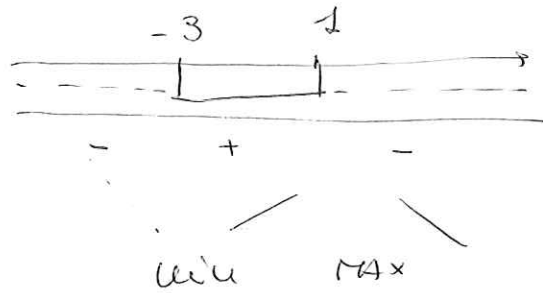
$$= e^{-\frac{1}{6}x^2} \left(1 - \frac{x^2}{3} - \frac{2x}{3}\right) = e^{-\frac{1}{6}x^2} \left(-\frac{x^2}{3} - \frac{2x}{3} + 1\right) \geq 0 \Rightarrow$$

$\geq 0 \quad \forall x \in D$

$$\Rightarrow -\frac{x^2}{3} - \frac{2x}{3} + 1 \geq 0 \Rightarrow \frac{-x^2 - 2x + 3}{3} \geq 0 \quad \begin{cases} 3 > 0 \text{ sempre} \\ -x^2 - 2x + 3 \geq 0 \end{cases}$$

$$\Rightarrow -x^2 - 2x + 3 = 0 \Rightarrow x_{1/2} = \frac{+2 \pm \sqrt{4+12}}{-2} = \frac{+2 \pm \sqrt{16}}{-2} = \frac{+2 \pm 4}{-2} \quad \begin{cases} -3 \\ 1 \end{cases}$$

segno y'



$$x=1 \quad y = (1+2)e^{\frac{1}{6}} + 1 = 3,53$$

$\Psi(1; 3,53)$

$$x=-3 \text{ mínimo} \Rightarrow y = (-3+2)e^{\frac{-1}{6}(-3)^2} + 1 = -e^{\frac{-9}{6}} + 1 = 0,77 \quad m(-3; 0,77)$$

$$\Rightarrow y = (-3+2)e^{\frac{-1}{6}(-3)^2} + 1 = (-1)e^{\frac{-9}{6}} + 1 = \underbrace{-e^{\frac{-3}{2}}}_{-0,22} + 1 = 0,77$$

$m(-3; 0,77)$ mínimo

$x=1$ max

$$y = (1+2)e^{\frac{1}{6}(1)^2} + 1 = \underbrace{3e^{\frac{1}{6}}}_{2,53} + 1 = 3,53$$

$M(1; 3,53)$ máximo

DERIVADA SEGUNDA

$$y'' = e^{\frac{-1}{6}x^2} \cdot \left(-\frac{1}{3}x\right) \cdot \left(-\frac{x^2}{3} - \frac{2}{3}x + 1\right) + e^{\frac{-1}{6}x^2} \left(-\frac{2}{3}x - \frac{2}{3}\right) =$$

$$= e^{\frac{-1}{6}x^2} \left[\left(-\frac{x}{3}\right) \cdot \left(-\frac{x^2}{3} - \frac{2}{3}x + 1\right) - \frac{2}{3}x - \frac{2}{3} \right] =$$

$$= e^{\frac{-x^2}{6}} \left(+\frac{x^3}{9} + \frac{2}{9}x^2 - \frac{x}{3} - \frac{2}{3}x - \frac{2}{3} \right) = e^{\frac{-x^2}{6}} \left(\frac{x^3}{9} + \frac{2}{9}x^2 - x - \frac{2}{3} \right) > 0 \quad \forall x \in \mathbb{R}$$

$$\Rightarrow \frac{x^3}{9} + \frac{2}{9}x^2 - x - \frac{2}{3} > 0 \Rightarrow \frac{x^3 + 2x^2 - 9x - 6}{9} > 0$$

$$\Rightarrow x^3 + 2x^2 - 9x - 6 > 0$$